International Journal of Theoretical Physics, Vol. 31, No. 2, 1992

Topological Geometrodynamics and the Solar Neutrino Problem

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Received May 13, 1991

A solution of the solar neutrino problem based on certain differences between T(opological) G(eometro) D(ynamics) and the standard model of the electroweak interactions is proposed. First, TGD predicts the existence of a right-handed neutrino inert with respect to ordinary electroweak interactions. Second, the generalization of the massless Dirac equation contains terms mixing different M^4 chiralities, unlike the ordinary massless Dirac equation. This and the observation of anticorrelations of the solar neutrino flux with sunspot number suggest that solar neutrinos are transformed to right-handed neutrinos on the convective zone of the Sun. Third, the compactness of CP_2 implies topological field quantization: space-time decomposes into regions, topological field quanta, characterized by a handful of vacuum quantum numbers. In particular, there are topological obstructions for the smooth global imbeddings of magnetic fields and the decomposition of the solar magnetic field into flux tubes is predicted. Finally, every electromagnetically neutral mass distribution is accompanied by a long-range Z^0 vacuum field. If the vacuum quantum numbers inside the flux tubes of the solar magnetic field are considerably smaller than in the normal phase, the Z^0 electric force becomes strong and implies Thomas precession for the spin of the left-handed component of the neutrino. As a consequence, left-handed neutrinos are transformed to right-handed ones and the process is irreversible, since right-handed neutrinos do not couple to Z^0.

1. TOPOLOGICAL GEOMETRODYNAMICS-INSPIRED SOLUTION TO SOLAR NEUTRINO PROBLEM

The solar neutrino problem (Davis *et al.*, 1988; Hirata *et al.*, 1986) (the average neutrino fluxes are smaller than predicted by the standard model of electroweak interactions) suggests that something might be wrong with the standard model of the electroweak interactions. It has recently turned out (Hirata *et al.*, 1986) that there is an additional puzzling feature related to

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0020-7748/92/0200-0245$06.50/0

the solar neutrinos: the fluxes observed in Homestake (Davis *et al.*, 1988) and Kamiokande (Hirata *et al.*, 1986) are different (about one-third and one-half of the predicted flux, respectively) and in Homestake the neutrino flux seems to anticorrelate with the appearance of sunspots, unlike in Kamiokande.

A second potential problem of the standard model is the anomalous production of e^+e^- pairs observed in heavy ion collisions (Chodos, 1987). The so-called *leptopion hypothesis* stating that there exist *bound states of color excited leptons* with mass of the order 1–2 MeV (Pitkänen, 1990*a*) provides a possible explanation for the anomalous production of e^+e^- pairs (see Pitkänen (1990*a*) and the preceding paper in the present issue).

Leptopion exchange implies a new weak interaction between leptons at low energies and this interaction might provide the explanation for the Homestake–Kamiokande puzzle (Davis *et al.*, 1988; Hirata *et al.*, 1986). The argument described in detail the accompanying paper goes as follows:

(a) Topological geometrodynamics [TGD; for the most recent review of TGD see Pitkänen (1990*b*)] predicts the existence of a *right-handed neutrino* inert with respect to standard electroweak interactions.

(b) *Part of the solar neutrinos are transformed to right-handed neutrinos* in the solar convective zone: this should provide the solution of the solar neutrino puzzle.

(c) *Right-handed neutrinos are observed in Kamiokande* through their scattering from ordinary leptons via *leptopion exchange*. In Homestake neutrino detection is based on the neutrino–nucleon scattering and since the leptopion does not couple to nucleons, right-handed neutrinos remain unobserved. Using as an input the Homestake flux, one obtains the correct prediction for the average Kamiokande flux and the anticorrelation with sunspots is predicted to be weaker, although not totally absent.

In this paper we shall consider a TGD-inspired solution for the *solar neutrino puzzle*. The model is based on certain general features distinguishing between TGD and conventional field concepts, which we list first.

(a) TGD predicts the *existence of a right-handed neutrino* and the TGD counterpart of the massless Dirac operator causes the *mixing of M^4 chiralities*, unlike the ordinary massless Dirac operator.

(b) CP_2 geometry implies what might be called *topological field quantization*: space-time decomposes into regions characterized by a handful of vacuum quantum numbers. This phenomenon might provide a universal mechanism for the generation of spatial and temporal structures. At astrophysical length scales space-time corresponds to the large-vacuum-quantum-number limit of TGD. At shorter length scales also small vacuum quantum numbers are possible: superfluids and superconductors provide possible candidates for the low-vacuum-quantum-number limit of TGD.

(c) The concept of induced gauge field implies that all electromagnetically neutral mass distributions are accompanied by *long-ranged vacuum Z^0 fields*: at the low-vacuum-quantum-number limit the Z^0 force becomes strong and dominates over gravitation.

The TGD-inspired solution of the solar neutrino problem boils down to a model for the *transformation of the right-handed neutrinos to left-handed ones in the solar convective zone* and is based on the following assumptions.

(a) The solar magnetic field consists of *discrete flux tubes*: a discrete flux tube structure results from the topological obstructions caused by CP_2 geometry for smooth imbeddings of electromagnetic fields and is a special case of topological field quantization.

(b) The vacuum quantum numbers associated with magnetic flux tubes are considerably smaller than for the ordinary vacuum, so that the vacuum carries a *strong Kähler field* [corresponding to the $U(1)$ gauge field of the standard electroweak model] and therefore also the Z^0 field, which is considerably stronger than the gravitational field. Besides the Z^0 magnetic field, a strong Z^0 electric field is present and plays a key role in the proposed mechanism.

(c) *Massive left-handed neutrinos are accelerated in the strong Z^0 electric field* of the magnetic flux tube parallel to the surface of the Sun. The Z^0 field *deflects neutrinos and accelerates them to higher energies.*

(d) The Z^0 magnetic field required to cause chirality flipping by the Z^0 magnetic moment interaction is unrealistically large. On the other hand, the acceleration in the Z^0 electric field causes *Thomas precession*, which is rapid at relativistic energies, and leads to the spin flip transforming a left-handed neutrino to a right-handed one. Since *right-handed neutrinos do not couple to the Z^0 field*, the process is *irreversible*, so that all neutrinos passing through a magnetic flux tube are transformed to right-handed ones.

In the following we shall give a brief review of relevant TGD concepts and construct a model for the $\nu_R \rightarrow \nu_L$ transformation. Calculational details are left to the Appendix.

2. SHORT REVIEW OF TGD

In order to construct a model for the chirality flipping of solar neutrinos, one needs some additional ideas of TGD (Pitkänen, 1990*b*) and in the following a brief review of relevant ideas is given.

1. The assumption that *physical space-times are representable as surfaces* of the space $H = M_+^4 \times CP_2$, where M_+^4 denotes the interior of the future light cone of Minkowski space and CP_2 is complex projective space of two complex dimensions, leads to a natural *geometrization of the gauge field concept* (Pitkänen, 1990*b*): electroweak gauge potentials are identifiable

as the components of the induced spinor connection, that is, projections of the CP_2 spinor connection to the surface representing space-time:

$$A_\alpha = A_k \partial_\alpha s^k \tag{1}$$

The geometrical meaning of the induction procedure is clear: parallel translation on X^4 is carried out using the spinor connection of H. In Pitkänen (1990*b*) it is shown that the components of the CP_2 generalized spinor connection are indeed identifiable as electroweak gauge potentials and that the correct coupling structure is obtained.

2. The induction procedure applies to the *spinor structure* of H, too. The spinors of X^4 are spinors of H and gamma matrices of X^4 are obtained as projections of H gamma matrices

$$\Gamma_\alpha = \Gamma_k \partial_\alpha h^k = \gamma_k(M^4) \otimes 1 \partial_\alpha m^k + \gamma_5 \otimes \Gamma_k(CP_2) \partial_\alpha s^k \tag{2}$$

As a consequence, a natural geometric interpretation for electroweak spin is obtained. A crucial feature differentiating between the TGD spinor structure and the ordinary spinor structure is that the CP_2 part of the gamma matrices connects different M^4 chiralities to each other. As a consequence, the generalized vector current $\bar{\Psi}\Gamma^\alpha\Psi$ contains a term coupling different M^4 chiralities to each other. This suggests that the induction procedure provides a *geometric description for the breaking of chiral invariance.*

The generalization of the massless Dirac equation is obtained by varying the massless Dirac action for induced spinors and metric and reads

$$\Gamma^\alpha D_\alpha \Psi = -\tfrac{1}{2}(D_\alpha \Gamma^\alpha)\Psi \tag{3}$$

A new feature is the *mixing of different M^4 chiralities* resulting from the fact that induced gamma matrices are linear superpositions of M^4 and CP_2 gamma matrices. This effect makes possible the mixing of chiralities of *massless particle without magnetic moment* and forms the basis for the TGD explanation of the $\nu_L \to \nu_R$ transformation. A second new feature is the appearance of a "mass term" on the right-hand side resulting from noncovariant constancy of the gamma matrices.

3. *Topological field quantization* is a very general phenomenon differentiating between TGD and ordinary gauge field concepts. The general space-time surface decomposes into regions characterized by four vacuum quantum numbers $\Lambda = (\omega_1, \omega_2, n_1, n_2)$, which are related to the space-time

dependence of the phase angles Ψ and Φ associated with the two complex coordinates of CP_2 (see Figure 1). For electromagnetically neutral space-times an additional integer-valued vacuum quantum number m, which we call the fractal quantum number, emerges.

Topological field quantization provides a possible first-principles explanation for the generation of spatial and temporal structures: in fact, a possible definition of a macroscopic subsystem is as a topological field quantum. One application of this phenomenon is the decomposition of a magnetic field into flux tubes, which takes place unless one allows edges of space-time (discontinuities in the derivatives of CP_2 coordinates as functions of space-time coordinates). The sizes of the topological field quanta depend on the values of the vacuum quantum numbers and are small (large) at the small (large)-vacuum-quantum-number limit of TGD.

The ordinary vacuum at astrophysical length scales must correspond to the high-vacuum-quantum-number limit of TGD. Superfluids and superconductors and in general the phases exhibiting macroscopic quantum effects are good candidates for the small-vacuum-quantum-number limit of TGD. In the case of the solar magnetic field the vacuum quantum numbers must be considerably smaller than in ordinary vacuum in order to explain the observed sizes of these structures naturally.

4. The concept of *configuration space* (Pitkänen, 1990*b*) plays a central role in the formulation of quantum TGD (Pitkänen, 1990*b*). Configuration space $C(H)$ consists of *all 3-surfaces of space H* (all manifold topologies and also singular manifold topologies intermediate between two manifold topologies are allowed).

The basic principle in configuration space geometrization is the requirement of $Diff^4$ *invariance*: in order to realize the action of $Diff^4$ in configuration space, the definition of the configuration space metric must somehow associate a unique space-time surface to a given 3-surface. The geometrization of the configuration space relies on the concept of the Kähler function

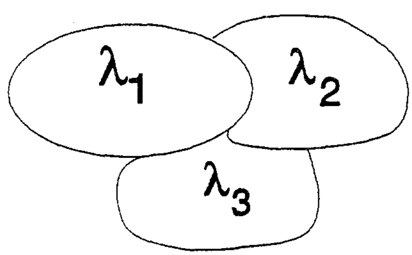

Fig. 1. Decomposition of space-time into "topological field quanta."

$K(X^3)$, which defines *Kähler geometry* in $C(H)$ via the formula

$$ds^2 = \partial_k \partial_{\bar{l}} K \, dz^k \, d\bar{z}^l \tag{4}$$

The value of the *Kähler function* for a given 3-surface X^3 is defined as the absolute minimum of the so-called Kähler action in the set of all 4-surfaces $X^4 \subset H$ containing X^3 as submanifold,

$$K(X^3) = \mathrm{Min}\{S_K(X^4) \,|\, X^3 \subset X^4\} \tag{5}$$

where the Kähler action is defined as the Abelian YM action associated with the projection $J_{\alpha\beta}$ of the CP_2 Kähler form to the 4-surface

$$S_K(X^4) = (1/16\pi\alpha_K) \int_{X^4} J^{\alpha\beta} J_{\alpha\beta} \sqrt{g} \, d^4x + \text{boundary term} \tag{6}$$

The metric of X^4 appearing in the formula is obtained by induction from the metric of H and the parameter $\alpha_K \simeq 28\alpha_{\mathrm{em}}/27 \simeq 1/137$ is called the Kähler coupling.

What is important is that the *definition of the Kähler function associates a unique space-time to a given* 3-*surface* and one can say that configuration space geometry defines what might be called classical physics. Second, the minimization of the Kähler action implies that the values of the time derivatives $\partial_t h^k(x)$ of H coordinates at X^3 and therefore of canonical momenta are determined by the minimization conditions, so that Bohr-type quantization rules result and the space-time surface can be regarded as a *generalized Bohr orbit*. This might provide an explanation for the quantization of masses, electric charge, etc., at the classical level. Third, the minimization of the Kähler action favors space-time surfaces with Kähler electric fields giving a negative contribution to the Kähler action (Kähler magnetic fields give a positive contribution).

5. The study of the macroscopic limit of the theory (Pitkänen, 1990*b*) shows that massive bodies are necessarily Kähler charged. This implies the existence of a *new* $U(1)$ *interaction mediated by the Kähler field.* Since the Kähler field corresponds to the $U(1)$ gauge field of the standard model, it gives rise to *a long-range* Z^0 *vacuum field* in case of electromagnetically neutral space-times and causes the *interaction of left-handed neutrinos with the vacuum*, while the interaction caused by the exchange of Z^0 quanta remains short-ranged.

The order of magnitude for the Kähler charge of macroscopic bodies is given by $Q_K \simeq M/\omega_1$. The requirement that this *interaction is weaker than the gravitational force* implies that the Z^0 charge of the particle at long distances is, apart from a numerical constant, equal to the mass of the particle divided by the Planck mass: $Q_K = \varepsilon_1 M/m_{\mathrm{Planck}}$, where $\varepsilon_1 \leq 1$, so that one has $\omega_1 \simeq m_{\text{Planck}}$. At the *elementary particle length scale the Kähler charge possesses its quantized value*, which varies in sign and magnitude for various particle species: at long length scales, charge renormalization makes the Kähler (and Z^0) charge small.

One consequence is that *moving matter creates the Kähler magnetic field*: $\nabla \times B_K = \varepsilon_1 10^{-19} N\beta$, where $N \equiv \rho/m_p$ denotes the nucleon density and β denotes the velocity field. The interaction of the solar neutrinos with Z^0 electric fields associated with the magnetic structures of the Sun plays a key role in the proposed explanation of $\nu_L \rightarrow \nu_R$ transformation.

6. Free particles are identified as 3-surfaces and *boundary components* of 3-surface are identified as the *carriers of elementary particle quantum numbers* (Pitkänen, 1990*b*) (the identification is obtained as a generalization of the string model, where string ends, "partons," are carriers of quark quantum numbers). As a consequence of this identification, one obtains a *topological explanation for family replication phenomena*: different fermion families correspond to different boundary topologies (sphere, torus, etc.). In Pitkänen (1990*b*) an argument explaining why *the number of light fermion families is three* is presented. The assumption implies that the generalized massless Dirac equation on the boundary component of a particlelike 4-surface provides a natural semiclassical model for the spin degrees of freedom of elementary particles.

7. The so-called *CP_2-type extremals provide the TGD model for a free elementary particle* (Pitkänen, 1990*b*), as a 3-surface with size of order the Planck length propagating with the velocity of light. The so-called vacuum CP_2 solutions are extremals of the Kähler action having four-dimensional CP_2 projection and defined by the following conditions:

$$m^k = f^k(s), \qquad m_{kl}(df^k/ds)(df^l/ds) = 0 \tag{7}$$

Here s is an arbitrary function of CP_2 coordinates and the condition states that the surface has as its M^4 projection an arbitrary *lightlike curve*. The induced metric and Kähler structure are identical to those of CP_2, so that one might call these surfaces "warped" CP_2's. As a special case one obtains the solution describing the propagation of a massless particle along a lightlike geodesic:

$$m^0 = m^3 = f(s), \qquad m^1 = \text{const}, \qquad m^2 = \text{const} \tag{8}$$

For various reasons these can be identified as the *Higgs = 0 phase* of the theory (Pitkänen, 1990*b*). The *counterparts of the Feynman diagrams* of the ordinary quantum field theory are obtained as topological sums

$CP_2\#CP_2\cdots\#CP_2$ of CP_2's (see Figure 2) by taking "warped" CP_2's and by gluing them together so that each line of the Feynman diagram corresponds to a "warped" CP_2 with one-dimensional M^4 projection and 3-particle vertices correspond to the regions where different CP_2's are glued together. The boundary components carrying elementary particle quantum numbers are obtained by drilling holes in each CP_2.

CP_2 extremals are not as such absolute minima of the Kähler action: in fact, the action is positive and given by $S_K=\pi/8\alpha_K$, but this does not make them unphysical. The large positive action implies a large value of the vacuum functional: $\exp[K(CP_2)]\simeq 10^{19}$ for the adopted value of the parameter α_K and therefore the probability that a particle corresponds to a CP_2-type boundary component is 10^{38} times larger than the probability that it corresponds to "hole" in surrounding 3-space. As a consequence, CP_2-type solutions should give an extremely good approximate classical model of an elementary particle.

In Pitkänen (1990*b*) we have given an argument explaining the *elementary particle mass scale* based on the idea that a CP_2 solution correspond to a Higgs$=0$ phase and a "hole" in background 3-space corresponds to an excitation with mass squared of the order of the Planck mass squared. The observed mass squared is the average value of the mass squared and is of the order of $\langle m^2\rangle\simeq p(\text{hole})/G\simeq 10^{-38}/G$.

8. The *Higgs mechanism* can be understood as a consequence of the so-called *topological condensation*, which means that particlelike 3-surfaces are glued to the background 3-surface possessing macroscopic size (formation of a topological sum).

(a) The minimization of the Kähler action implies the *generation of a radial Kähler electric field and Kähler charge*. At the elementary particle level the particle mass can be identified as the energy of its Kähler electric and magnetic fields. The value of the Kähler charge is, apart from a numerical constant of order, one, given by $Q_k\simeq M/\omega_1\equiv\varepsilon_1 M/m_{\text{Planck}}$. In the ordinary

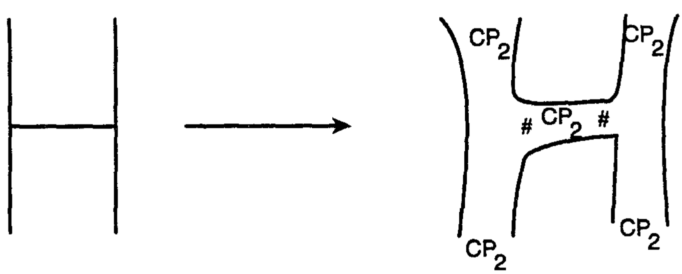

Fig. 2. Connected sums of CP_2-type extremals as "Feynman diagrams with lines thickened to four-manifolds."

vacuum the Kähler force must be weaker than the gravitational interaction, so that the value of ω_1 must be of the order of the Planck mass, so that one has $\varepsilon_1 \le 1$.

(b) In the topological condensation of a CP_2-type extremal, the *motion along a lightlike geodesic is transformed to zitterbewegung* along a lightlike curve so that the center of mass remains at rest. In length scales much shorter than the size of the zitterbewegung orbit the particle looks like a massless particle in accordance with the idea that at sufficiently short length scales the Higgs$=0$ phase sets in. A natural but not necessary assumption for what follows is that the CP_2 projection of the cm of the boundary component is a geodesic line of CP_2.

The simplest model for the zitterbewegung orbit is as a *circle of radius* ρ_0: the boundary component of the CP_2-type extremal moves with the velocity of light along this circle. By a dimensional argument the *Kähler energy and therefore the rest mass of the particle is determined by the radius of this circle*: $m \propto 1/\rho_0$; so that the Compton wavelength of the particle corresponds to the radius of its zitterbewegung orbit. The upper bound $m \le 1$ eV for its mass implies that its zitterbewegung orbit has a radius larger than $\rho_0 \ge 10^{-7}$ m. Zitterbewegung generates a *Kähler magnetic moment* having an order of magnitude $\mu_K \propto \rho_0 \propto 1/m$. At long length scales renormalization effects make this magnetic moment small: $\mu_K \simeq 10^{-19}\varepsilon_1\rho_0$.

9. The description of *the Higgs mechanism at the spinorial level* goes as follows. At the field theory limit (Pitkänen, 1990*b*) elementary fermions (and also bosons) are described by second quantized free spinor fields restricted to the boundary components of the CP_2-type extremal satisfying the generalized Dirac equation.

(a) The Dirac equation for boundary spinors can be written in the form, where M^4 and CP_2 degrees of freedom are separated,

$$\Gamma^{\alpha}_{CP_2} D_\alpha \Psi + \tfrac{1}{2}(D_\alpha \Gamma^{\alpha}_{CP_2})\Psi = -\gamma_k \partial_\alpha m^k g^{\alpha\beta} D_\alpha \Psi - \tfrac{1}{2}\gamma_k (D_\alpha \partial_\beta m^k)\Psi \tag{9}$$

The left-hand side corresponds to the contribution of CP_2 gamma matrices to the gamma matrices of the boundary component. For lightlike M^4 orbits the M^4 metric does not contribute to the induced metric. The right-hand side corresponds to the M^4 contribution to the induced gamma matrices and is treated as a small perturbation (instead of CP_2 coordinates, M^4 coordinates are in the role of dynamical fields!). This means that CP_2 gamma matrices correspond to the chirality-preserving term in the Dirac equation and *the M^4 part of the induced gamma matrices plays the role of the chirality-mixing term.*

(b) There are two contributions causing chirality mixing. The first contribution corresponds to the M^4 contribution to the induced gamma matrices appearing in the $\Gamma^\alpha D_\alpha$ part of the Dirac operator. The contribution of

M^4 gamma matrices to induced gamma matrices is proportional to $O = \gamma_k \, df^k/ds$ and is *nilpotent* ($O^2=0$) for all lightlike curves [$m^k = f^k(s)$]. For lightlike geodesics [say $m^0 = m^3 = f(s)$] also the covariant derivatives of O are nilpotent since one has $D_\alpha O \propto O$. The second contribution comes from *the M^4 part of the second fundamental form* and is given by the expression $M = \gamma_k g^{\alpha\beta} D_\alpha \partial_\beta m^k$. This contribution is not a lightlike vector except for lightlike geodesics: in this case the two contributions are proportional to each other: $M \propto O$.

(c) *Lightlike geodesics corresponds to the Higgs zero phase*, as is easy to see. First, for lightlike geodesics the operator O, the covariant derivatives of O, and M are nilpotent and one can assume that physical spinors are annihilated by this operator: $\Psi = O\chi$. The condition is obviously *equivalent to the ordinary massless Dirac equation* and implies that both M^4 matrices as well as the M^4 metric disappear totally from the Dirac equation.

A more general situation corresponds to a small deformation of a lightlike geodesic with the property that the M^4 part of the second fundamental form remains a lightlike vector and *the gauge field part of the chirality-mixing term becomes nonvanishing*. In this case chirality mixing takes place for all fermions *except the right-handed neutrino*, which has no coupling to gauge fields. The peculiar feature is that *mixing is not symmetric*: the left-handed neutrino transforms to the right-handed one, but not vice versa. This suggests that *massivation takes place for all leptons except the right-handed neutrino*. This is in accordance with the fact that the right-handed (but not the left-handed) neutrino has a vanishing electroweak hypercharge, which we have identified as the Kähler charge. The peculiar unidirectional property of the mixing plays an important role in the proposed explanation for the transformation of the solar neutrinos to right-handed neutrinos.

(d) For zitterbewegung orbits the covariant derivatives of the operator O as well as the operator M fail to be nilpotent. Therefore it is not possible to avoid chirality mixing ($\Psi = O\chi$ is no longer a good solution ansatz) and fermions become massive. Chirality mixing is caused by two terms: the term $O \partial_\alpha s g^{\alpha\beta} D_\beta$ and the term M, which also couples to the right-handed neutrino.

3. MODEL FOR THE MIXING OF SOLAR NEUTRINOS

The transformation of left-handed neutrinos to right-handed ones inside the convective zone of the Sun explains both the Kamiokande and Homestake results if the leptopion hypothesis is accepted. The remaining task is to construct a *model explaining the observed mixing ratio*. After a considerable number of trials and errors, we have ended up with a scenario which explains the observed anticorrelation with the sunspots and is consistent with the basic ideas of TGD.

(a) There is considerable observational evidence that the solar magnetic field consists of *discrete flux-tube-like structures* (Zirin, 1988).

(b) *Magnetic flux tubes differ from an ordinary vacuum* in that the transformation to right-handed neutrinos takes place very rapidly inside them.

(c) *The mixing phenomenon is irreversible*: the transformation ν_R to ν_L does not take place appreciably, so that all left-handed neutrinos passing through magnetic flux tubes are transformed to right-handed ones.

Topological field quantization provides a natural TGD explanation for the flux tube structure. The smallness of the topological vacuum quantum numbers of the magnetic flux tube implies the presence of a strong Z^0 field, which couples to the left-handed neutrinos only and causes Thomas precession, leading to an irreversible chirality flip.

From these assumptions it follows that the expression for the fraction of right-handed neutrinos from the total neutrino flux is essentially the *fractional area occupied by the projections of the flux tubes of the solar magnetic field to the surface of the Sun* at the equator, where the observed solar neutrinos originate:

$$P(\nu_R) = P(A) \tag{10}$$

Anticorrelation implies that this fractional area should vary in the range $[1/4, 4/5]$. The visible vertical parts of the magnetic flux tubes associated with sunspots do certainly correspond to a considerably smaller fractional area than 3/4 (Zirin, 1988). The contribution of the sunspots is not, however, the only contribution to the fractional area. The observations indicate that flux tubes with all sizes down to the observational limit of the order of 10^4 m are possible (Zirin, 1988). Furthermore, the tangential parts of the magnetic flux tubes must also be taken into account and since all tangential flux tubes in the convective zone contribute to the projection, the fractional area can well be of the required magnitude.

The assumption that the solar magnetic field consists of discrete flux tubes plays a key role in the model and has a strong empirical basis (Zirin, 1988). In a separate paper we construct a model for the magnetic structure of the Sun in terms of flux tubes and the model predicts the *correct order of magnitude for the period of the solar magnetic cycle* [11 years instead of 10^{10} years(!) predicted by a naive magnetohydrodynamic estimate (Zirin, 1988). We include in the Appendix a model for the cylindrically symmetric flux tube, which hydrodynamically corresponds to a *helical vortex*: it is shown that magnetic flux tubes very probably differ from the ordinary vacuum in that the value of the parameter ε_1 is large for them, so that the gravitational force is negligible as compared to the vacuum Z^0 force.

Furthermore, the flux tubes carry radial Kähler electric field of the order of $E_K \simeq B_K/\beta_{rot}$. It is these two features, which imply the rapid transformation of neutrinos to right handed ones.

The following more detailed model for solar neutrino mixing justifies the remaining assumptions of the proposed model.

(a) A long-range Kähler field implies a long-range Z^0 field, which can be derived from the electromagnetic neutrality condition (see Appendix):

$$Z^0 = -3J/\sin^2(\theta_W) \tag{11}$$

A left-handed neutrino interacts with the Z^0 field of the magnetic flux tube via its Z^0 charge, which is, apart from a numerical factor, equal to the Kähler charge $Q_K = \varepsilon_1 m_\nu/m_{\mathrm{Planck}}$.

The order of magnitude for the Kähler magnetic field of the magnetic flux tube modeled as a hydrodynamical vortex (see Appendix) is obtained by a dimensional argument from the equation $\nabla \times B_K = \varepsilon_1 10^{-19} N\beta$ and the order of magnitude of the radial Kähler electric field is obtained from $B_K \simeq E_K \beta_{rot}$, so that one obtains

$$B_K \simeq 10^{-19} \varepsilon_1 N \beta_{rot} \rho_v, \qquad E_K \simeq 10^{-19} \varepsilon_1 N \rho_v \tag{12}$$

where $N \simeq 10^{29}/\mathrm{m}^3$ is a typical number density of nucleons in the convective zone, and $\beta_{rot} \simeq 10^{-5}$ is an estimate for the rotational velocity of the vortex. The order of magnitude for B_K is about 10^{-6} T for the smallest flux tubes of thickness $\rho_v \simeq 10^4$ m and 10^{-3} T for the largest flux tubes with a radius of order 10^7 m.

(b) The *acceleration of neutrinos in the radial Z^0 electric field* of the Kähler magnetic flux tube provides the most effective mechanism for $\nu_L \to \nu_R$ transformation. The study of the relativistic equations of motion $\{d[v/(1-v^2)^{1/2}]/dt = Q_{Z^0} E_{Z^0}/m_\nu\}$ shows that the kinetic energy gained by the neutrino when traversing the conservative Z^0 field of a cylindrically symmetric horizontal flux tube is given by

$$\Delta E_{kin} \simeq \delta(q_{Z^0} Z^0) \tag{13}$$

where Z^0 denotes the potential of the Z^0 field. If the Z^0 field is attractive for the left-handed neutrino, it gains kinetic energy in the Z^0 field of the magnetic flux tube and is deflected. Deflection is considerable when the magnitude of the Z^0 potential energy is comparable to the kinetic energy of the neutrino.

(c) *Acceleration in the Z^0 electric field causes spin flip via Thomas precession*, which is a purely kinematic effect. The angular velocity of Thomas precession is given in the laboratory frame (Jackson, 1962) by

$$\bar{\omega}_T = \frac{\gamma^2}{\gamma+1}\,\bar{v}\times\bar{a}, \qquad \gamma = \frac{1}{(1-v^2)^{1/2}} \tag{14}$$

where $\bar{v}$ and $\bar{a}$ denote the velocity and the acceleration, respectively, suffered by the spinning particle.

In the present case the acceleration is given by

$$\bar{a} = Q_{Z^0}\bar{E}_{Z^0}/m(\nu) \tag{15}$$

The radial background Z^0 field of the Sun is in the same direction as the neutrino velocity and causes no precession. The radial Z^0 electric field associated with the helical vortices can, however, cause spin flipping and the angular velocity is indeed orthogonal to the neutrino velocity. Q_{Z^0} is the spin average of the Z^0 charge and vanishes for purely right-handed neutrinos.

An order-of-magnitude estimate for the precession velocity is obtained using (i) the order-of-magnitude estimate for the Kähler magnetic field of the helical vortex [$B_K \geq \varepsilon_1 10^{-6}$ T and $B_Z = (3/\sin^2\theta_W)B_K \geq \varepsilon_1 10^{-5}$ T; see Appendix], (ii) the order-of-magnitude estimate for the Z^0 charge of the neutrino $Q_{Z^0} \simeq 10^{-19}\varepsilon_1 m\nu/m_p$ (m_p is the proton mass), and (iii) the upper bound $m(\nu)$ for the mass of the neutrino. The result reads

$$\omega_T \simeq 10^{-5}\varepsilon_1^2\,\frac{E}{\text{MeV}}\,\frac{\text{eV}}{m(\nu)}\Big/\text{sec} \tag{16}$$

The precession rate is proportional to the energy of the neutrino and inversely proportional to the mass of the neutrino.

There are two alternative ways to achieve a spin flip time much smaller than the time spent in traversing the magnetic flux tube. Either (i) *the mass of the neutrino is small,*

$$m(\nu) \leq 10^{-10}\ \text{eV} \tag{17}$$

or (ii) the neutrino mass is of the order of $m(\nu) \simeq 1$ eV (we shall show that supernova observations give some support for this assumption) and the value of the parameter ε_1 is large: the condition

$$\varepsilon_1 \geq 10^5 \tag{18}$$

guarantees that the spin flip takes place also for the thinnest observed flux tubes (Zirin, 1988) with radius of the order of 10^4 m. The study of the model for magnetic flux tubes (see Appendix) of the Sun, however, suggests that the value of the parameter $\varepsilon_1 = m_{\text{Planck}}/\omega_1$ is considerably larger than that inside the magnetic flux tubes.

(d) The Z^0 field *does not couple to right-handed neutrinos* and this feature explains the *irreversibility of* $\nu \rightarrow \nu_R$ *transformation.* A natural assumption is that the Z^0 charge is the spin average of the Z^0 charge and since it vanishes for the purely right-handed polarization, the *neutrino propagates as a massless, purely right-handed neutrino after having reached completely right-handed polarization.*

In the TGD picture of the Higgs mechanism the transformation to a right-handed neutrino corresponds to the transformation of the *zitterbewegung orbit to lightlike geodesics*: this explains classically why the vanishing coupling to the Z^0 field becomes trivial.

At the spinorial level the transformation looks as follows. During the deformation of the zitterbewegung orbit to a lightlike geodesic the M^4 part of the fundamental form becomes a lightlike vector. If the gauge field contribution to the chirality mixing changes more slowly to a lightlike vector field, it begins to dominate chirality mixing, with the result that *chirality mixing becomes unidirectional* and the spinor field becomes essentially right-handed in the final state and to a good approximation all neutrinos traveling through a magnetic flux tube are transformed to right-handed neutrinos.

(e) For sufficiently high Z^0 fields the acceleration of the neutrinos in the Z^0 field causes the *depletion of low-energy neutrinos from the neutrino flux.* If all low-energy neutrinos are transformed to right-handed ones in all magnetic flux tubes, then the fraction of the observed low-energy flux from the predicted by standard model is just $P(A)$. On the other hand, if there exist very thin flux tubes causing only acceleration but no appreciable transformation to right-handed neutrinos, then the fraction of low-energy neutrinos can become smaller than $P(A)$.

If one requires that the kinetic energy gained in the Z^0 electric field is of the same order of magnitude as the initial energy of the neutrino $[q_{Z^0} E_{Z^0} L \geq E(\nu)]$, then, using the estimates $E_\nu \simeq 1$ MeV for the neutrino energy and $L \simeq 10^6$ m for the thickness of the thinnest flux tubes, one obtains the lower bound

$$\varepsilon_1 \geq 10^6 \tag{19}$$

so that the depletion of the low-energy part of the neutrino spectrum is a possibility to be considered seriously unless the mass of the neutrino is very small. It should be noticed that the effect of a Z^0 field of this magnitude on electrons is completely negligible.

4. COMMENTS

Some comments about the proposed scenario are in order.

1. *TGD concepts of gauge field and spinor structure* play central roles in the explanation. What makes the proposed model attractive is that it avoids two basic difficulties of standard models.

(a) The properties of CP_2 spinors imply that mixing takes place in one direction only. If the mixing takes place in both directions, the mixing rate must be of the order of magnitude of the thickness of the convective zone. For larger mixing rates coherence is lost and the fractions of left-handed and right-handed neutrinos become equal. The difficult question to answer is why the fluctuations in the strength of the magnetic field and in the thickness of the convective zone do not spoil the coherence.

(b) The magnetic moment needed to produce the observed effective mixing angle tends to become too large unless the value of solar magnetic fields is assumed to be of the order of 10 T: observations (Zirin, 1988) suggest that the field strengths are of the order of 10^{-1} T.

2. The observations about the *neutrino flux coming from SN1987A* provide an independent experimental indication about the correctness of the proposed model. Besides the main burst of neutrinos, the detection of a neutrino burst before the main burst was reported (Aglietta *et al.*, 1987). A TGD explanation for the phenomenon is obvious: *the anomalous neutrino burst corresponds to left-handed neutrinos which have transformed to right-handed ones inside the supernova* and the main burst corresponds to left-handed neutrinos. From the known distance of SN1987A and from the time difference between bursts (Pitkänen, 1990*b*) one can estimate the mass of the neutrino and obtain the estimate $m_\nu \simeq 1$ eV.

3. The abnormally small value of ω_1 implies that gravitational interaction becomes negligible as compared with the Z^0 force. The appearance of so-called *prominences* on the surface of the Sun is a phenomenon which indeed seems to defy the gravitational force and could be regarded as a manifestation of a phase with small ω_1. We have also proposed (Pitkänen, 1990*b*) that strong Kähler fields near the surface of a supernova might be the cause of *supernova explosions.*

4. The following argument suggests that *superconductors and superfluids* should correspond to regions of spacetime with a small value of ω_1. The quantization of magnetic flux and velocity circulation are explained naturally provided the electromagnetic field and the Kähler field are equal, apart from a sign factor. This in turn implies that the Kähler field must be strong and the value of the parameter ω_1 is of the order of the elementary particle mass. If so, then *superconductors or superfluids could transform left-handed neutrinos to right-handed ones* almost instantaneously. One can also compare the *propagation of light through superconductors or superfluids and ordinary matter.* Since the velocity for free propagation of light with respect to M^4 time is of the order of $(1-R^2\omega_1^2/4)^{1/2}$ the velocity for the free propagation of light in superconductors should be considerably larger.

APPENDIX

A.1. Spacetime Surfaces with Vanishing Electromagnetic Fields

Using the so-called Eguchi–Hanson coordinates (r, Θ, Ψ, Φ) for CP_2 (Pitkänen, 1990*b*) (Ψ and Φ are essentially the phase angles of CP_2 complex coordinates), the expression of the Kähler form reads

$$
\begin{aligned}
&J = (r^2/F^2)\, dr \wedge (d\Psi + \cos\Theta\, d\Phi) + (r^2/2F) \sin\Theta\, d\Theta \wedge d\Phi \\
&F = 1 + r^2
\end{aligned}
\tag{A1}
$$

The general expression for the electromagnetic field is derived in the Appendix of Pitkänen (1990*b*) and the explicit expression reads

$$
\begin{aligned}
F_{\mathrm{em}} &= (3+2p)(r^2/F^2)\, dr \wedge (d\Psi + \cos\Theta\, d\Phi) \\
&\quad + (3+p)(r^2/2F) \sin\Theta\, d\Theta \wedge d\Phi \\
p &= \sin^2(\Theta_W)
\end{aligned}
\tag{A2}
$$

where Θ_W denotes the Weinberg angle. The vanishing of the electromagnetic fields is guaranteed when the conditions

$$
\begin{gathered}
\Psi = k\Phi \\
(3+2p)(r^2/F^2)(dr/d\Theta)(k+\cos\Theta) + (3+p)(r^2/(F) \sin\Theta = 0
\end{gathered}
\tag{A3}
$$

hold. The conditions imply that the CP_2 projection of electromagnetically neutral space-times is 2-dimensional. Solving the differential equation, one obtains

$$
\begin{aligned}
r &= \tan(X) \\
X &= (\varepsilon/2) \ln[|(k+\cos\Theta)/C|] \\
\varepsilon &= (3+p)/(3+2p)
\end{aligned}
\tag{A4}
$$

where C is an integration constant. r belongs to the correct range provided the value of the quantity X satisfies the conditions $X \in [m\pi, (2m+1)\pi/2]$, where the integer m labels the branch of the arctangent used. This implies

$$
C \exp(2m\pi/\varepsilon) \le u + k \le C \exp[(2m+1)\pi/\varepsilon] \tag{A5}
$$

The lower bound corresponds to the $r=\infty$ and the upper bound to the $r=0$ surface.

As such, the general expression for u in terms of r is defined on the region of space-time bounded by the $r=\infty$ and $r=0$ surfaces, which corresponds to a definite range of the variable $u=\cos(\Theta)$. The values of u which correspond to the $r=0$ and $r=\infty$ surfaces are given by

$$\begin{aligned} u_0 &= C\exp(2m\pi/\varepsilon)-k \\ u_1 &= C\exp[(2m+1)\pi/\varepsilon]-k \end{aligned} \tag{A6}$$

As a consequence, space-time decomposes into regions characterized by different values of the vacuum parameters. At $r=\infty$ surfaces, n_2, ω_2, and m can change, since all values of Ψ correspond to the same point of CP_2: at $r=0$ surfaces also n_1 and ω_1 can change, since all values of Φ correspond to the same point of CP_2, too.

This implies what might be called *topological field quantization*; since in general it is not possible to find *smooth global imbedding* for, say, constant magnetic field. Although global imbedding exists, it decomposes into regions with different values of vacuum parameters and the coordinate u in general possesses a discontinuous derivative at the $r=0$ and $r=\infty$ surfaces. The only way *to avoid edges of spacetime* is to allow *topological field quantization*, so that the field decomposes into topological field quanta, which can be regarded as structurally stable units of the gauge field.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + n_2\phi + \text{Fourier expansion} \\ \Phi &= \omega_1 m^0 + n_1\phi + \text{Fourier expansion} \end{aligned} \tag{A7}$$

m^0 and ϕ denote time and angle coordinates associated with cylindrical M^4 coordinates, so that one has $k=\omega_2/\omega_1$. Regions of space-time with given values of the vacuum parameters ω_i and n_i and of m and C are bounded by the $r=0$ and $r=\infty$ surfaces.

The vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \omega_2/n_2 - \omega_1/n_1 = 0 \tag{A8}$$

is satisfied. In particular, the ratio ω_2/ω_1 is a rational number for electromagnetically neutral regions of space-time. The *change of the parameters n_1 and n_2 (ω_1 and ω_2) in general generates a magnetic field* and therefore we shall refer to these integers as *magnetic* (*electric*) *quantum numbers*.

The expressions for the Kähler form and the Z^0 field of electromagnetically neutral space-time will be needed in the sequel and are given by

$$\begin{aligned} J &= -[p/2(3+p)]\sin^2 X\, du \wedge d\Phi \\ Z^0 &= -(3/p)J \end{aligned} \tag{A9}$$

When space-time is electromagnetically neutral the generation of long-range Kähler electric fields implied by the minimization of the Kähler action implies also the generation of *vacuum* Z^0 *fields*: this effect differentiates between TGD and the standard model of electroweak interactions. Notice that the components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field.

A.2. Helical Vortex As a Model for a Magnetic Flux Tube

The model of the magnetic flux tube as a helical vortex is based on the following physical picture.

(a) *The velocity field of the vortex serves as a source of the Kähler magnetic field,*

$$\nabla \times \bar{B}_{\mathrm{K}} = 10^{-19}\varepsilon_1 N\bar{v} \tag{A10}$$

where $N \equiv \rho_m/m_p$ denotes nucleon density and $K = \varepsilon_1 10^{-19}$ describes the strength of the Kähler force. $\varepsilon_1 \leq 1$ measures the relative strength of the Kähler and gravitational forces. For the gravitational interaction to dominate over the Kähler force the condition $\varepsilon_1 \leq 1$ must hold.

(b) The *magnetic field* is generated when the integers n_i change so that their ratio differs from the value $n_1/n_2 = \omega_1/\omega_2$ guaranteeing electromagnetic neutrality. This mechanism implies that the *magnetic and Kähler magnetic fields are parallel to each other.*

(c) *Magnetohydrodynamic stability conditions* are satisfied if the magnetic field of the sunspot is parallel to the electric current, so that the *Lorentz force vanishes*: $\nabla \times \bar{B} = \bar{v} \propto \bar{B}_{\mathrm{em}}$ (Zirin, 1988). If a magnetic field is generated by changing the values of the magnetic quantum numbers n_1 and n_2, then Kähler magnetic and magnetic fields are parallel, so that also the *Kähler magnetic and velocity fields are parallel*:

$$\bar{B}_{\mathrm{K}} \propto \bar{v} \tag{A11}$$

Helical vortices are the simplest objects allowing this kind of structure and cylindrical symmetry fixes the structure of the helical vortex almost completely.

The helical vortex possesses cylindrical symmetry in the sense that the Kähler magnetic field and the velocity field have only z and ϕ components, which depend on the cylindrical coordinate ρ only, so that one has

$$\begin{aligned}
\Phi &= \omega_1 t + k_1 z + n_1 \phi \\
\Psi &= k\Phi = \omega_2 t + k_2 z + n_2 \phi \\
r &= \tan X(u) \\
X(u) &= \ln[(k+u)/C]\varepsilon/2 \\
u &= u(\rho)
\end{aligned} \tag{A12}$$

The relationship between the velocity field and the Kähler magnetic field is dictated by the condition that matter flow serves as a source of the Kähler magnetic field.

The expressions for the nonvanishing components of the induced Kähler magnetic field are

$$\begin{aligned} B_z^{\mathrm{K}} &= -[p/2(3+p)]n_1 \sin^2 X\, \partial_\rho u/\rho \\ B_\phi^{\mathrm{K}} &= -[p/2(3+p)]k_z \sin^2 X\, \partial_\rho u \rho \end{aligned} \tag{A13}$$

The requirement $\nabla \times \bar{B}^{\mathrm{K}} \propto \bar{B}^{\mathrm{K}}$ implies the condition

$$\partial_\rho B_z^{\mathrm{K}}/\partial_\rho B_\phi^{\mathrm{K}} = -(B_\phi^{\mathrm{K}}/\rho^2 B_z^{\mathrm{K}}) \tag{A14}$$

Using the explicit representation for the induced field, one obtains the differential equation

$$\begin{aligned} \partial_\rho Y &= \{[1-(\rho/\rho_1)^2]/[1+(\rho/\rho_1)^2]\rho\}Y \\ Y &= \sin^2 X\, \partial_\rho u \\ \rho_1 &= n_1/k_z^1 \end{aligned} \tag{A15}$$

which gives

$$\begin{aligned} \partial_\rho Y &= \{[1-(\rho/\rho_1)^2]/[1+(\rho/\rho_1)^2]\rho\}Y \\ Y &= \sin^2 X\, \partial_\rho u \end{aligned} \tag{A16}$$

Integrating this equation, one obtains

$$\begin{aligned} B_z^{\mathrm{K}} &= -[p/2(3+p)]n_1/\{[1+(\rho/\rho_1)^2]\rho_0^2\} \\ B_\phi^{\mathrm{K}} &= -[p/2(3+p)]k_z^1\rho^2/\{[1+(\rho/\rho_1)^2]\rho_0^2\} \end{aligned} \tag{A17}$$

where ρ_0 is an integration constant possessing the dimension of length.

The magnitudes of the velocity components β_z and β_ϕ are

$$\begin{aligned} \beta_z &= (2k_z^1/NK\rho_0^2)\,\frac{p}{2(3+p)} - 1 + \left(\frac{\rho^2}{\rho^1}\right) \\ \beta_\phi &= (\rho/\rho_1)\beta_z \end{aligned} \tag{A18}$$

Stability requirements for helical vortices (Chandrasekhar, 1961) suggest that the value of n_1/k_z^1 is of the same order as the critical radius. Notice that the vortex rotates like a rigid body near the z axis and that the longitudinal velocity is also approximately constant near the z axis.

The above-described imbedding of the helical Kähler magnetic field fails at the *critical radius* $\rho = \rho_{\mathrm{cr}}$, which corresponds to the value of $r = \infty$. The imbedding can be continued for larger values of ρ by allowing the change of the parameters m, n_2, and ω_2 at the $\rho=\rho_{cr}$ surface (all values of Ψ correspond to the same point at the $r=\infty$ surfaces), but the radial derivative of the variable u becomes discontinuous on the surface $\rho=\rho_{cr}$, which therefore becomes an *edge of space-time*. This suggests strongly the occurrence of *flux quantization* in the sense that the region inside ρ_{cr} serves as a *structurally stable unit of the magnetic field.*

The expression for the critical radius in the present case is obtained from the condition $r=\infty$ and reads

$$\begin{aligned}\rho_{cr}&=\rho_1\{\exp[4(\rho_0/\rho_1)^2(u_0+k)\exp(-2\pi m/\varepsilon)X_0]-1\}^{1/2}\\&\simeq 2\rho_0\exp(-m\pi\varepsilon)[(u_0+k)X_0]^{1/2}\\X_0&=[1/(1+\varepsilon^2)][(2+\varepsilon^2)\exp(\pi/\varepsilon)+\varepsilon^2]\end{aligned}\tag{A19}$$

where we have assumed that the value of the exponent is small. We shall soon find that the assumption is physically well founded. Notice that the critical radius depends extremely sensitively on the value of the "*fractal*" *quantum number m* and that the critical radii are related by a *multiple of the discrete scaling transformation* in the approximation used.

An essential point is that the vortex also carries a radial Kähler electric field: the magnitude of this field is given by

$$|E_K|=|B_\phi^K|(\omega_1\rho/n_1)\tag{A20}$$

The presence of this field plays an essential role in the proposed explanation for the solar neutrino puzzle.

A.3. Estimates for the Vacuum Parameters of Magnetic Flux Tubes

Consider next the values of the various *vacuum parameters* appearing in the embedding of the helical vortex.

(a) From the requirement that *the gravitational interaction is stronger than the Kähler force in long length scales* one obtains $\omega_1\simeq m_{\text{Planck}}$ (Pitkänen, 1990*b*). Lorentz invariance implies that the value of k_z^1 is given by

$$k_z^1\simeq\omega_1\beta_z\simeq\beta_z/\sqrt{G}\tag{A21}$$

It turns out that this estimate of ω_1 does not hold true inside magnetic flux tubes if one wants to solve the solar neutrino problem in the proposed manner.

(b) The requirement that *the angular momentum density is of the correct order of magnitude* gives an estimate for the value of the parameter n_1. The expression of the conserved angular momentum current in the z direction is

$$J^\alpha=T^{\alpha\beta}\partial_\beta m^k m_{kl}j^l\tag{A22}$$

where j^k denotes the vector field associated with infinitesimal rotation and $T^{\alpha\beta}$ denotes the energy-momentum tensor. For the angular momentum density one obtains in cylindrical M^4 coordinates for X^4 the expression

$$J^t = T^{t\phi}\rho^2, \qquad T^{\alpha\beta} = (1/16\pi G)G^{\alpha\beta} \tag{A23}$$

where the second equation is Einstein's equation. If the *contribution of the CP_2 curvature to the curvature tensor is not dominating*, the leading-order contribution to $G^{t\phi} = R^{t\phi} - g^{t\phi}R/2$ comes from the nonvanishing of the metric component $g_{t\phi}$:

$$g_{t\phi} = s^{\text{eff}}_{\Phi\Phi}\omega_1 n_1 = -(R^2/4)[\cos^2(X)(k+u)^2 + 1 - u^2]\sin^2(X)\omega_1 n \tag{A24}$$

and one obtains the order-of-magnitude estimate

$$J^t \simeq -T^{tt}g_{t\phi} \simeq \rho_m R^2 \omega_1 n_1/4 \tag{A25}$$

In order to obtain the correct order of magnitude for the angular momentum density associated with rotational flow, one must have

$$R^2\omega_1 n_1/4 \simeq \rho\beta(\rho) \tag{A26}$$

which implies

$$n_1 \simeq (L/R^2\omega_1)\beta \simeq (L/R)\varepsilon_1 \tag{A27}$$

where L and β are the typical scale and velocity associated with the flow. If L is taken to be the radius of the vortex ($L \simeq 10^7$ m) and $\beta_\phi \simeq 10^{-5}$ the rotation velocity of the vortex, one obtains $n_1 \simeq 10^{35}\varepsilon_1$. If L is taken to be the radius of the Sun and β the rotation velocity of the Sun, the value of n_1 is about 100 times larger: $n_1 \simeq 10^{36}\varepsilon_1$.

If the *Kähler field is strong* as compared to the gravitational field, the dominating contribution to $G^{t\phi}$ comes from the contribution of CP_2 curvature to $R^{t\phi}$ and is proportional to the quantity $J^t_\rho J^{\rho\phi}$: in this case the previous estimate no longer holds, and one obtains the estimate

$$n_1/\omega_1 \simeq \beta L \tag{A28}$$

Since the Kähler field is strong inside sunspots, one must use this estimate for n_1/ω_1 and one obtains the estimate $E_K \simeq B_K/\beta_{\text{rot}}$ using the relationship between the Kähler magnetic and electric fields.

(c) *An estimate for the parameter* ρ_0 is obtained by substituting the estimate of k_z in the general expression of β_z at the z axis and one obtains the condition

$$\rho_0 \simeq 10^{19}(p/3+p)/(\sqrt{G}\,N\varepsilon_1)^{1/2} \simeq (1/\varepsilon_1)^{1/2}10^{11}\ \text{m} \tag{A29}$$

where the estimate $N \simeq 10^{30}/m^3$ for the nucleon density has been used.

(d) An estimate for the value of the fractal quantum number m is obtained from the condition that the exponent appearing in the expression of the critical radius is small:

$$4(\rho_0/\rho_1)^2 \exp(-2m\pi\varepsilon)[(u_0+k)X_0] \ll 1 \tag{A30}$$

Since one has $\rho_0 \simeq (1/\varepsilon_1)^{1/2} 10^{11}$ m and $\rho_1 \simeq \rho_{cr} \simeq 10^6$ m, one obtains an order-of-magnitude estimate $\exp(-2m\pi/\varepsilon) \ll 10^{-10}\varepsilon_1/(u_0+k)$, so that the value of m must be rather large unless the value of the parameter $u_0+k=u_0+n_2/n_1$ is very small or the value of ε_1 is sufficiently large: the value $\varepsilon_1 \geq 10^5$ suggested by the neutrino mixing model implies that m is of order 2: a rather natural-looking value, unlike the large values implied by $\varepsilon_1 \simeq 1$.

(e) If the magnetic field is generated by the change of n_1 so that the condition $\omega_1/\omega_2 = n_1/n_2$ ceases to hold, one obtains the following approximate expression for the magnetic field at the z-axis:

$$B_z^{\text{em}} \simeq \Delta n_1(3+p)/\rho_0^2 \tag{A31}$$

The requirement that the field is of the order of $B_{\text{em}} = 10^3$ G gives the estimate $\delta n_1 \simeq 10^{36}/\varepsilon_1$, so that the change of n_1 is given by $\Delta n_1/n_1 \simeq 10/\varepsilon_1^2$ and is larger than one unless the value of the parameter ε_1 is large. Too large a value of Δn_1 is not in accordance with the idea that electromagnetic fields are generated as small perturbations of an electromagnetically neutral background, so that ε_1 should be considerably larger than one: $\varepsilon_1 \gg 1$ in turn implies that the Kähler force dominates over gravitation. The large value of ε_1 ($\varepsilon_1 \geq 10^6$) in turn explains the transformation of solar neutrinos to right-handed ones.

ACKNOWLEDGMENTS

We thank J. Maalampi and J. Peltoniemi for enlightening discussions concerning various aspects of the solar neutrino problem.

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